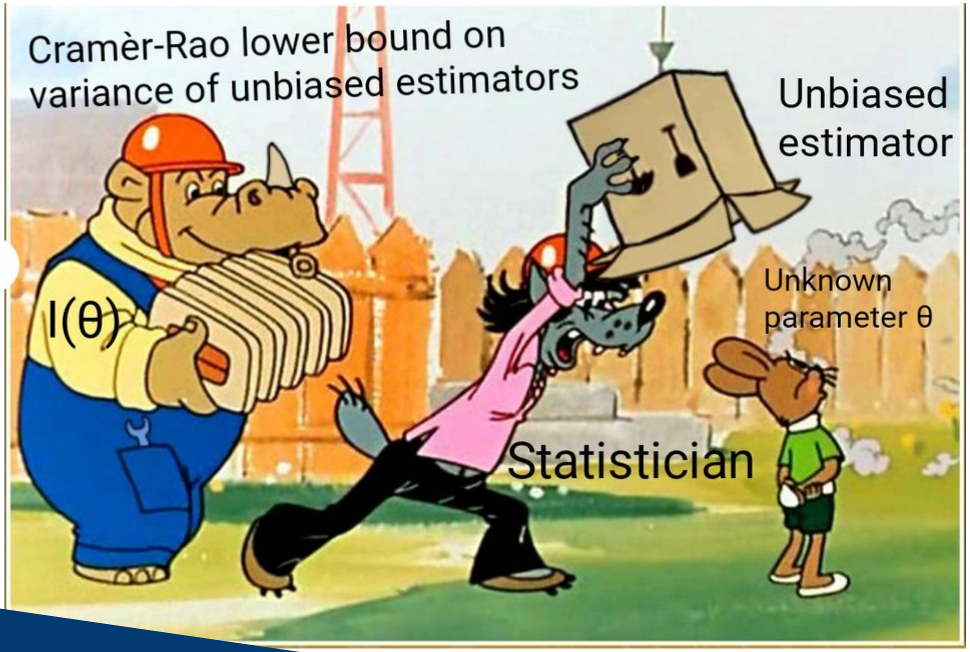
Stat 135 Lab 5

GSI: Yutong Wang
Feb 21, 2020

Berkeley
UNIVERSITY OF CALIFORNIA

To-do Today

1. Minimal sufficient statistic
2. Sufficiency
3. Fisher information and Cramer-Rao lower bound
4. Quiz time (45 min)



Berkeley
UNIVERSITY OF CALIFORNIA

Minimal Sufficient Statistic (Rice 8.10.47)

The Pareto distribution has been used in economics as a model for a density function with a slowly decaying tail:

$$f(x|x_0, \theta) = \theta x_0^\theta x^{-\theta-1}, \qquad x \geqslant x_0, \theta > 1$$

Assume that $x_0 > 0$ is given and that $X_1, \dots, X_n$ is an i.i.d. sample.

(a) Find the MoM estimate of θ.

(b) Find the MLE of θ.

(c) Find the asymptotic variance of the MLE.

(d) Find a sufficient statistic for θ.

(e) Find a minimal sufficient statistic.

Berkeley
UNIVERSITY OF CALIFORNIA

Sufficiency

Suppose you observe $X_1, \ldots, X_n \overset{\text{i.i.d.}}{\sim}$ Bernoulli(p). Recall that if $0 \leqslant p \leqslant 1$, then $0 \leqslant p(1-p) \leqslant 1/4$.

(a) What is the MLE of p?

(b) Is $\sum_{i=1}^{n} X_i$ sufficient for p? Show that X_1 is not sufficient but $X_1, \ldots, X_n$ (i.e., the sample itself) is sufficient.

(c) Find the estimator with minimum variance among all unbiased estimators. Explain why your choice has this property.

(d) Suppose you want to estimate $\theta = p(1-p)$. Find an unbiased estimator $\hat{\theta}$ of θ that depends only on $\overline{X}_n$.

Berkeley
UNIVERSITY OF CALIFORNIA

FI and CR lower bound

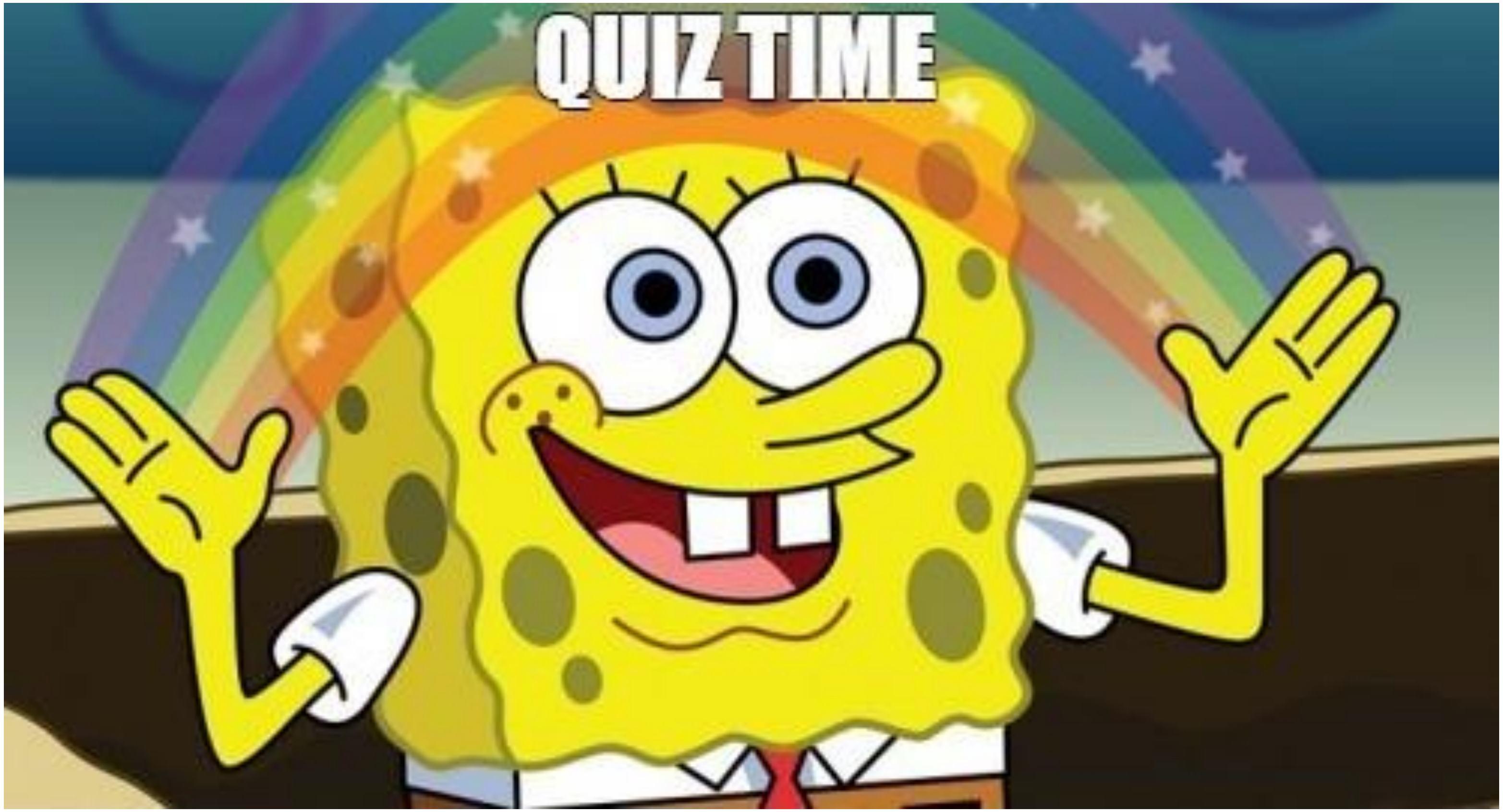
Consider an i.i.d. sample $Y_1, \ldots, Y_n$ with pdf $f(y) = \theta e^{-y\theta}$ for $\theta > 0$ and $y > 0$.

(a) Derive the likelihood function. Where in your derivation are you using independence?

(b) Use the factorisation theorem to find a sufficient statistic for θ. Show your work.

(c) Find the MLE for θ.

(d) Find the MLE for $\text{Var}(Y) = 1/\theta^2$. State which property of MLEs you are using.


QUIZ TIME